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PLASTIC EXTRUSION OF A SQUARE SHAPE FROM THE CIRCULAR TUBE

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Abstract *A theoretical analysis of the extrusion of a circular tube into a square shape is developed. A computational model for the shell is proposed in order to derive a closed-form solution for the axial load necessary to perform the extrusion. The most significant deformation components are discussed.*
The plastic deformations of the tube are caused by the change in the curvature in the circumferential direction. In the axial direction, the deformation is associated to non linear effects. The shear effects are present but they have not been taken into account because they represent a small fraction of the total energy involved in the process.
A closed-form solution is derived for the force necessary to extrude the tube. This solution is presented in a dimensionless form relating the force to the geometric and mechanical parameters of the process.
The solution is compared to the results of an axis-symmetric crushing of a tube which is related to the present problem. Application of the solution to the design of energy absorbing devices is discussed.
Key Words: *Plasticity, Extrusion, Shell, Energy Absorbing Device*

1. INTRODUCTION

The study of devices for energy absorption specially during impact situations has been addressed by several researchers. In the case of tube crushing we can point out important contributions from Norman Jones *et al.*(1984) and Wierzbicki *et al.*(1983). For the case of tube splitting, Stronge *et al.*(1983) have developed simplified models which allowed for an analytical expression of the forces involved.

For our case another mechanism of dissipation is proposed: the extrusion of a circular tube. A tube of radius R is extruded against a die of total length l in such a way to produce a square tube, having a corner radius r_0. Fig. 1 shows the process.

The configuration of the tube changes continuously along the length of the die. We assume that the deformations are restricted to the length of the die, and because of that the region is called region of deformation.

The configuration of the tube in the zone of plastic deformation involves lines where the

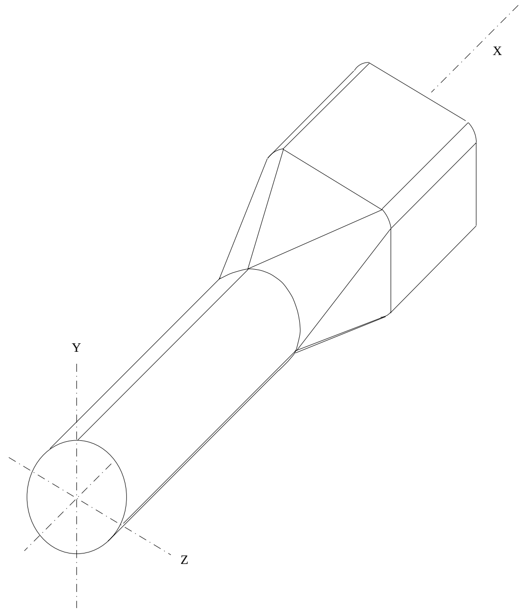

Figure 1: The extrusion process of a square shape from the circular tube

curvature is not continuous. Consequently, we can say that the hinge lines separate two regions of continuous curvature.

The rate of dissipation of the internal energy consists mainly of the following components: a) energy resulting from the hinge propagation ; b) energy resulting form a continuous change of curvature ; c) axial deformations introduced by changing of the line elements. In addition, the deformations in axial and circumferential directions are considered to be independent. Thus, the problem in both directions is considered to be uncoupled.

The statement of the global equilibrium is expressed by the balance equation between the rate of the work of external forces and the rate of the internal work. In symbols:

$$F_{ext}v = \frac{dE_{int}}{d\tau} + F_f v \tag{1}$$

where F_{ext} represents the external force applied to the tube, v represents the process velocity and $\frac{dE_{int}}{d\tau}$ is the rate of internal work and τ is the time. The statement of equilibrium should also include a term relating the external dissipation of energy due to the friction forces developed in the deforming zone through the die surfaces, for the non-ideal case. Consequently, F_f represents the friction forces.

2. MECHANISM OF DEFORMATION

We assume a rigid-perfectly plastic idealization of the material with a stress-strain rate relation represented analytically by:

$$\sigma = \sigma_0 sign\ \dot{\epsilon} \tag{2}$$

where σ_0 is the yield stress and $\dot{\epsilon}$ is the strain rate. The rigid-perfectly plastic model is convenient in working with problems involving large plastic deformations of metal structures. The elastic strains are a small fraction of the total strains and can be disregarded.

The assumptions adopted for the present modeling can be summarized as: a) For the rings, the model involves continuous deformation and localized deformations at the hinge line ; b) The bending moments are assumed $M_{\theta\theta} = M_0$ (fully plastic bending moments, i.e $M_0 = \frac{\sigma_0 t^2}{4}$) ; c) The axial forces acting on the generators in the deforming region are equal to $N_{xx} = N_0$, i.e, $N_0 = \sigma_0 t$, where the stresses are uniform throughout the wall thickness t ; d)The internal energy dissipated due to the shearing and twisting are not taken into account ; e) The inextensibility of the rings is adopted $\epsilon_{\theta\theta} = 0$; f) The axial bending moments are assumed negligible $M_{xx} = 0$

3. GENERAL EQUATIONS

For *a shell element* , the variations of the energy density e_{int} may be expressed by:

$$e_{int} = \int \{M_{\alpha\beta}\delta\kappa_{\alpha\beta} + N_{\alpha\beta}\delta\epsilon_{\alpha\beta}\}dA \tag{3}$$

where: $M_{\alpha\beta}$ are the bending moments, $N_{\alpha\beta}$ are the in-plane forces, $\kappa_{\alpha\beta}$ represents the variation of curvature during the process , $\epsilon_{\alpha\beta}$ contain the strain components and dA the element of area.

4. GEOMETRY AND DISPLACEMENTS

The symmetry of the deformations, as observable in Fig. 1 permit us to restrict the analysis to one eighth of the circumference. Fig 2 shows the deformed configuration of a quarter of circumference superimposed to the initial sectional configuration. The deformed configuration changes along the axial position.

For the proposed geometry of deformation, we can express the current position of any point and construct a displacement function $w = w(x, \theta)$. The displacement function is constructed observing:

- The inextensibility of the rings ;
- The elements are displaced in such a way to produce two different curvatures, depending on the original angular position with respect to the hinge point.

These postulates have been used in the construction of the displacement function in terms of the coordinates and geometrical parameters (R, r_0) as sketched in Fig 2. In the figure, H denotes the *hinge point*, (P, Q) are arbitrary points with locations before and after the hinge point. Their current location after deformation is (P_c, Q_c). θ_h is the angular position of the hinge and θ' is the angular position relative to the hinge position. The figure is self-explanatory.

5. DISPLACEMENTS

The displacement function w can be expressed in terms of the initial angular position θ, and the magnitude of the residual radius r_0, and it takes the form:

$$\begin{aligned}\frac{w^2}{r_0^2} = &\{\frac{x}{l}\frac{\pi}{4}(R - r_0) + [R - \frac{x}{l}(R - r_0)\cos(\theta') - R\cos(\theta)\}^2 + \\ &\{\frac{x}{l}\frac{\pi}{4}(R - r_0) + [R - \frac{x}{l}(R - r_0)\sin(\theta') - R\sin(\theta)\}^2\end{aligned} \tag{4}$$

for points satisfying the condition:

$$0 \leq \theta \leq \theta_h$$

Similarly, an expression for those points whose original angular position θ are larger than the hinge angular coordinate θ_h can be found. For those points, the displacements are described

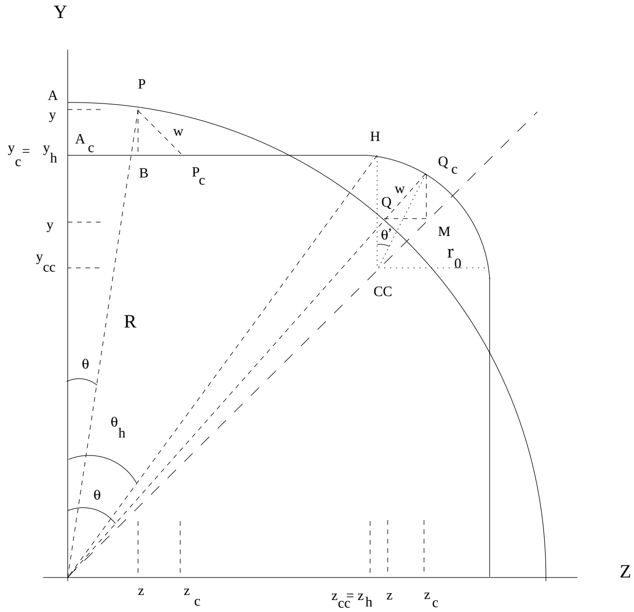

Figure 2: The geometrical parameters used to characterize the displacement function

by:

$$\frac{w^2}{r_0^2} = \{\frac{x}{l}\frac{\pi}{4}(\frac{R}{r_0} - 1) + [\frac{R}{r_0} - \frac{x}{l}(\frac{R}{r_0} - 1)\cos(\theta') - \frac{R}{r_0}\cos(\theta)\}^2 + \{\frac{x}{l}\frac{\pi}{4}(\frac{R}{r_0} - 1) + [\frac{R}{r_0} - \frac{x}{l}(\frac{R}{r_0} - 1)\sin(\theta') - \frac{R}{r_0}\sin(\theta)\}^2 \tag{5}$$

and they satisfy the condition:

$$\theta_h \leq \theta \leq \frac{\pi}{4}$$

The parameter θ' is defined by:

$$\theta' = \frac{R}{r(x)}(\theta - \theta_h)$$

where $r(x) = [R - \frac{x}{l}(R - r_0)]$ and θ_h corresponds to:

$$\theta_h = \frac{\frac{x}{l}\frac{\pi}{4}(R - r_0)}{R}$$

Fig 3 presents the displacements along the die in terms of the angular position for the case $\frac{R}{r_0} = 10$.

6. ENERGY IN THE DEFORMING ZONE

The computational model permit us to separate the energy calculations associated with the generators a part from the energy calculations associated with the rings.

6.1 Deformation of the rings

The energy dissipated during the deformation of the rings is computed in terms of the Dissipation Function, which involves two different processes.

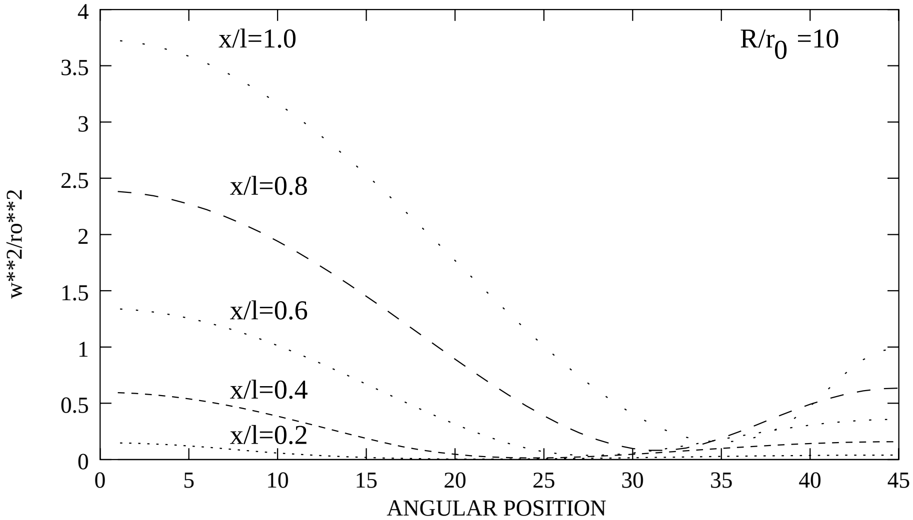

Figure 3: The displacement function of the generators

We can define a function to analyze the dissipation which occurs in the region of continuous dissipation. Thus, associated with the continuous dissipation region, we can construct the continuous dissipation function, here denoted as C_{df}, and define C_{df} by:

$$C_{df} = \int_0^1 M_0 l \kappa_{\theta\theta} d(\frac{x}{l})$$

The integration produces:

$$C_{df} = M_0 l \frac{-1}{(R - r_0)} \{ \frac{\theta}{\frac{\pi}{4}} + \ln[1 - \frac{\theta}{\frac{\pi}{4}}] \} \tag{6}$$

valid for the case:

$$0 \leq \theta \leq \frac{\pi}{4} \frac{(R - r_0)}{R}$$

Whereas for the rest of the eighth of the circle:

$$C_{df} = M_0 l \frac{-1}{R} \{ 1 + \frac{R}{(R - r_0)} \ln(\frac{r_0}{R}) \} \tag{7}$$

valid for the case:

$$\frac{\pi}{4} \frac{(R - r_0)}{R} \leq \theta \leq \frac{\pi}{4}$$

We say that the discontinuity in the curvature of the generators defines the dissipation of energy along a line, the hinge line. And different from the continuous dissipation which extends over a region, the hinge dissipation is localized. Thus, associated with the hinge dissipation, we can construct the hinge dissipation function, and denote it by H_{df}. *The hinge dissipation function* H_{df} is:

$$H_{df} = \int_0^1 M_0 \kappa_{\theta\theta} \delta(\frac{x}{l} - \frac{x}{l}|_\theta) d(\frac{x}{l})$$

where $\delta()$ corresponds to the Dirac Delta Function.

For the hinge dissipation function H_{df} we can write:

$$H_{df} = M_0 l \frac{1}{R} \{ \frac{\frac{\pi}{4}}{[\frac{\pi}{4} - \theta]} \} \quad (8)$$

valid for the case:

$$0 \leq \theta \leq \frac{\pi}{4} \frac{(R - r_0)}{R}$$

We can study the energy dissipated by each generator inside the deforming region plotting the continuous dissipation function C_{df} combined with the hinge dissipation function H_{df}. Fig4 shows the curves of functions involved and the total dissipation R_{df}, as the sum of C_{df} and H_{df}.

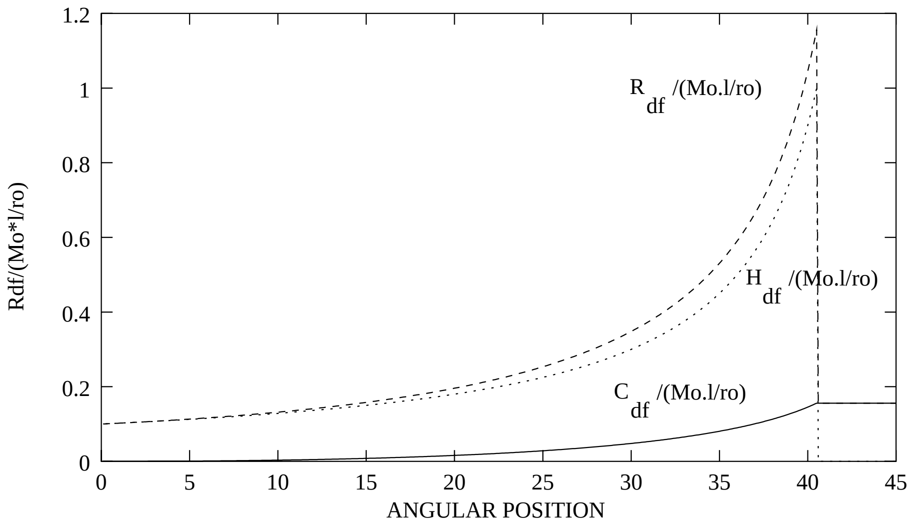

Figure 4: The components of the dissipation function of the rings.

6.2 Energy of continuous deformation

For each generator θ, at any axial position $\frac{x}{l}$, the function $\frac{d\theta}{ds}$ represents the change of the relative rotation that has been done over the section between the entrance position until the current position.Thus, computing first the work done over all sections inside the deforming zone for any generator, and taking into account all generators afterwards, we get:

$$W_{cont} = \int_0^{\frac{\pi}{4}} C_{df} d\theta$$

where M_0 represents the fully plastic bending moment of the sections and the subscript *cont* is used to emphasize the meaning of the expression.

Performed the integration we get the final result:

$$W_{cont} = \frac{1}{2} M_0 l \frac{\pi}{4} \frac{1}{R} \frac{(R - r_0)}{R} \quad (9)$$

that represents *the total energy dissipated* in the deforming zone through the continuous dissipation mechanism.

6.3 Energy of localized deformation

The other component of the dissipation process is present at the hinge points. Thus, the work done over each element depends on the value of the current curvature *immediately before* the axial position where the hinge appears. Not all generators will have a hinge point. The generators that have hinge point are those ones whose original angular coordinate θ is in the range:

$$0 \leq \theta \leq \frac{\pi}{4} \frac{(R - r_0)}{R}$$

In order to take into account the work done over all elements on the hinge line, and all generators, the integral has to be calculated:

$$W_{hge} = \int_0^{\theta_{max}} H_{df} d\theta$$

where the subscript *hge* has been used to remember us that this equation calculates the work dissipated through the hinge mechanism.

The result of the integration produces:

$$W_{hge} = M_0 l \frac{\pi}{4} \frac{1}{R} \ln(\frac{R}{r_0}) \tag{10}$$

6.4 Total energy dissipated

We can combine the partial contributions due to the continuous deformation and the hinge process. Thus, we can add the expression of Eq. 9 and Eq. 10 in order to get the total work. The energy dissipated for the complete ring can be expressed by:

$$\frac{\frac{W}{l}|_{ring} R}{\sigma_0 t^2} = \frac{\pi}{2} \{ \frac{1}{2}(1 - \frac{r_0}{R}) + \ln(\frac{R}{r_0}) \} \tag{11}$$

where we have taken care of the total circumference and we have substituted the value of the fully plastic bending moment M_0.

7. DEFORMATION OF THE GENERATORS

In order to calculate the force necessary for the extrusion, we need to describe the deformation of the generators in the deforming zone. We need to compute the energy dissipated during the extrusion. Since we assume that only direct tensile stresses are present, then shear deformations are not accounted for.

From the expression of the strain in the axial direction of deformation, and admitting no-stretching of the generators, the deformations become expressed only by the non-linear terms. Such approximation seems to be reasonable since there is the influence of the friction forces along the length of the die during the extrusion process.Thus:

$$\epsilon_{xx} = \frac{1}{2} [(\frac{\partial (y_c)}{\partial (x)})^2 + (\frac{\partial (z_c)}{\partial (x)})^2] \tag{12}$$

The corresponding values of the partial derivatives are:

$$\frac{\partial(y_c)}{\partial(x)} = \frac{r_0}{l}(\frac{R}{r_0} - 1)[(\frac{\pi}{4} - 1)] \tag{13}$$

being:

$$0 \leq \theta \leq \theta_h$$

For the remaining of the cross-section:

$$\frac{\partial(y_c)}{\partial(x)} = \frac{r_0}{l}(\frac{R}{r_0} - 1)[\frac{\pi}{4} - \cos(\theta') - \frac{\frac{R}{r_0}(\theta - \frac{\pi}{4})}{[\frac{R}{r_0} - \frac{x}{l}(\frac{R}{r_0} - 1)]\sin(\theta')}] \tag{14}$$

being:

$$\theta_h \leq \theta \leq \frac{\pi}{4}$$

Similarly for the other partial derivative:

$$\frac{\partial(z_c)}{\partial(x)} = 0 \tag{15}$$

for the case:

$$0 \leq \theta \leq \theta_h$$

and

$$\frac{\partial(y_c)}{\partial(x)} = \frac{r_0}{l}(\frac{R}{r_0} - 1)[(\frac{\pi}{4} - \sin(\theta') + \frac{\frac{R}{r_0}(\theta - \frac{\pi}{4})}{[\frac{R}{r_0} - \frac{x}{l}(\frac{R}{r_0} - 1)]\cos(\theta')} \tag{16}$$

for the case:

$$\theta h \leq \theta \leq \frac{\pi}{4}$$

The work done over the generators of one eighth of circumference along the die is expressed in a dimensionless form by:

$$\frac{W_{gen} l}{\sigma_0 t[r_0(\frac{R}{r_0} - 1)]^2} = \int_0^1 \int_0^{\frac{\pi}{4}} \frac{1}{2}[f^2(\theta', \frac{x}{l}, \frac{R}{r_0}) + g^2(\theta', \frac{x}{l}, \frac{R}{r_0})] d\theta d(\frac{x}{l}) \tag{17}$$

where (f, g) are dimensionless terms extracted from the strain measure.

The inner integral can be evaluated analytically and it is useful in the understanding of the dissipation along the die. The variation of inner integral in terms of the axial position for several reduction ratio $\frac{R}{ro}$ is presented in Fig 5. The integral of Eq. 17 can be found analytically and the numerical variation in terms of the reduction ratio $\frac{R}{ro} = \{10, 20, ..., 50\}$ is presented in Fig 6

7. CONCLUSION

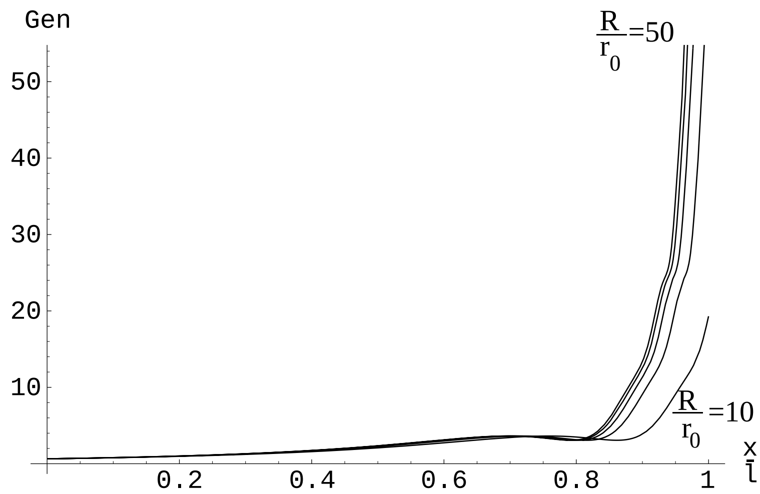

Figure 5: The energy dissipated for $\frac{R}{ro} = \{10, 20, ..50\}$

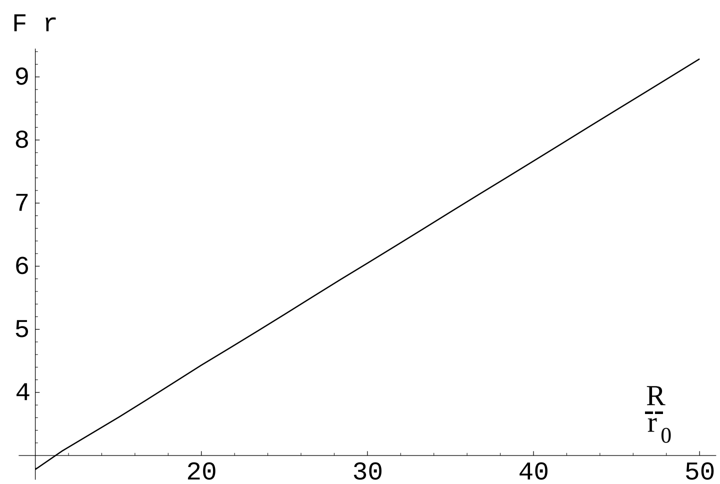

Figure 6: The energy dissipated in terms of the reduction ratio

The contribution from the generators and rings to the total energy dissipated is obtained from Eq. 11 and Eq. 17. The parameter l is determined via the minimization of the total energy, which afterwards is used to find the expression of the force required. The force depends on the reduction ratio. For small reduction ratios it can be approximated by:

$$\frac{F}{\sigma_0 t^2} = 2\pi^2\{\frac{1}{2}(1 - \frac{r_0}{R}) + \ln[\frac{R}{r_0}]\} \tag{18}$$

where F corresponds to the total force applied.

The mean crushing force of tubes developed by Wierzbicki *et al.* corresponds to:

$$\frac{F}{\sigma_0 t^2} = 9.56(\frac{C}{t})^{\frac{1}{3}}$$

where C represents the side of the square tube, t its thickness.

For the case of the dynamical mean crushing force of circular tubes, Abramowicz *et al.* obtained the expression:

$$\frac{F}{\sigma_0 t^2} = 31.33(\frac{R}{t})^{\frac{1}{3}}$$

We can conclude that the extrusion process is an intermediate process in terms of its capacity to absorb energy and it could be used efficiently as device for absorbing energy during impact. For other cases, additional information is available in the references.

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